Classical Quarterly **32** (ii) 312–322 (1982) *Printed in Great Britain*

THE ONE AND THE MANY: ARISTOTLE ON THE INDIVIDUATION OF NUMBERS*

In Book *K* of the *Metaphysics* Aristotle raises a problem about a very persistent concern of Greek philosophy, that of the relation between the one (τὸ ἕν) and the many (τὸ πλῆθος), but in a rather peculiar context. He asks: 'What on earth is it in virtue of which mathematical magnitudes are one? It is reasonable that things around us [i.e. sensible things] be one in virtue of [their] ψυχή or part of their ψυχή, or something else; otherwise there is not one but many, the thing is divided up. But [mathematical] objects are divisible and quantitative. What is it that makes them one and holds them together?' (1077a20–4).

It is not immediately apparent what Aristotle is asking for here. Alexander of Aphrodisias, in his commentary on the passage, illustrates the expression ἄλλῳ τινὶ εὐλόγῳ by the case of things glued or tied together,[1] and Aristotle does indeed speak of things which are glued or bound together as being 'accidentally' one elsewhere in the *Metaphysics* (*Δ*, 1015b34–1016a4). Annas, who follows Alexander's interpretation, remarks on the peculiarity of such a question but claims that it has force as a refusal 'to take anything on trust without proof which the platonist assumes about mathematical objects'. But if this were the case, then, as she points out, there is an obvious reply to such a trivial objection, for 'a platonist would say in irritation that the triangles he is talking about are not the sorts of things that can fall apart, as though they were bundles of sticks tied together with string'.[2] There are, in fact, good reasons why we should not take this view of Aristotle's question. The problem of what makes numbers 'one' is something that had also worried Plato in the *Theaetetus* (205e), and the way the problem is raised by Aristotle on a number of occasions in the *Metaphysics* (e.g. *A*, 991b22, 992a1; *Δ*, 1020b3; *H*, 1044a2–6) and in the *Physics* (e.g. *Δ*, 224a2–16) shows quite plainly that what he is primarily concerned with is a question about the individuation and identity of numbers, a problem which is raised when we ask what makes the number ten *one* number, for example.

He distinguishes elsewhere between the numbers that are counted in the thing in question, which we can call sensible numbers, and the numbers by which we count these, which we can call noetic numbers (*Ph.* *Δ*, 219b6–7). Noetic numbers are abstractions and are therefore quite distinct from Plato's noetic numbers (whether these be Forms or 'intermediate' numbers – cf. *Metaph.* *A*, 987b14 f.), since these have independent being whereas Aristotle's noetic numbers do not.[3] On the other hand, they are distinct from sensible numbers, which are numbers of sensible things and therefore not abstractions. For Aristotle, sensible numbers are (genetically) prior to

* This paper owes a great deal to those who have commented on earlier drafts, particularly Professor A. A. Long and two anonymous referees.

[1] cf. W. D. Ross, *Aristotle's Metaphysics* (2 vols. Oxford, 1953), ii. 414.

[2] J. Annas, *Aristotle's Metaphysics Books M and N* (Oxford, 1977), pp. 145–6.

[3] It might be considered helpful, in the light of this, to employ different terms for Plato's and Aristotle's 'noetic' numbers, but I think there are advantages in maintaining the same term for the two. The main advantage derives from the fact that Aristotle, in his discussion of numbers, is often explicitly criticizing Plato with the aim of challenging Plato's conception of the ontological status of non-sensible numbers, rather than drawing a different distinction which cuts across Plato's. Their respective views of sensible numbers are essentially the same, Plato characterizing them as those numbers which have 'visible and tangible body' (*Rep.* 525d).

noetic numbers in that the latter are abstractions from the former; however, noetic numbers are (logically) prior to sensible numbers in that we require a knowledge of noetic numbers – minimally, in the form of what we would now call a recursive procedure for generating numbers in the proper order – for us to be able to count numbers of sensible things in the first place. The distinction between the two 'kinds' of number is paralleled in geometry, in the distinction between the shapes of sensible things and abstract (noetic) geometrical figures.

This difference between sensible and noetic numbers must be taken into account in asking what it is that makes numbers one. In the case of sensible numbers, we can ask what it is that makes a decad of sheep one decad, for example. Is the decad one in virtue of a kind to which each of the sheep belongs (and of which they are distinct instantiations), or in virtue of a resemblance that exists between each of the sheep (a resemblance which is logically prior to their being treated as being of the same kind), or in virtue of some unity which they derive from our ascribing a definite number to them? The problem is the familiar one of what it is that makes a one out of a many. But it is far from clear that Aristotle's usual type of example of this problem – an animal being many in that its constituent parts are many yet being one in virtue of its ψυχή – is of much help in the case of numbers. It is hard to think of a parallel unifying principle in the case of decads, and the problem is even more complicated when we turn to noetic numbers, since these are abstractions and it is unclear whether abstractions can even be 'ones'.

My purpose in this paper is to clarify the sense in which Aristotle considers numbers to be 'ones', and thereby to elucidate one of the more problematic aspects of his conception of mathematics. A rather important consideration that I hope will become clear in the course of the discussion is the difficulty of drawing parallels with modern debates in the philosophy of mathematics. Mathematics prior to Vieta's work on algebra in the late sixteenth century is very different from mathematics as we now understand it, particularly as regards arithmetic.[4] Nevertheless, while there may be no general systematic connection to be made, there are specific issues where comparison is helpful, and such comparisons will be introduced when they allow us to throw some light on Aristotle's conception of number.

I. GENERIC INDIVIDUATION

Aristotle's discussions of number occur almost exclusively in the context of discussions of Platonist accounts of number and, since this is particularly true of his treatments of what it is that makes numbers 'ones', it will be of considerable help in clarifying what is at issue if we can specify precisely the difference between Aristotle's view and that of the Platonists. There are two major differences, on the question of the ontological status of numbers and on the question of what the unity of numbers derives from. We shall return to the first question below but, briefly, on the Platonist conception numbers are independent abstract objects, and Aristotle finds this view quite incoherent (cf. *de An. Γ*, 432b15 f.; *Ph. B*, 193b31 f.; *Metaph. K*, 1061a28 f.). While both Plato and Aristotle consider that we must have a logically prior knowledge of noetic numbers if we are to be able to count numbers of sensible things, Aristotle, unlike Plato, refuses to assign any ontological priority to noetic numbers on this basis (cf. *Metaph. M*, 1077a14–20).

As far as the question of the unity of numbers is concerned, the Platonist view is that noetic numbers derive their unity from their genus. They are the kinds of things

[4] cf. J. Klein, *Greek Mathematical Thought and the Origin of Algebra* (Cambridge, Mass., 1968).

they are in virtue of the genus to which they belong, and it is in virtue of their belonging to this genus that they can be identified and individuated. The Platonist view depends crucially upon a classification of kinds (εἴδη) of numbers by a systematic ordering of their units (μονάδες). The most schematic such orderings are to be found in the writings of the neoplatonists, and particularly in Nicomachus' *Introductio Arithmetica*, where the primary kinds are given as the even and the odd, which are then subdivided into the even times the even and the odd times the even. Nicomachus then proceeds to other classifications determined by the nature of the sum or product of aliquot parts (yielding perfect, superabundant and deficient numbers), by ratio, and by figuration (yielding triangular, quadratic and oblong numbers). There is considerable precedent for this procedure in Plato, who presents the 'first kinds' as the odd and the even (*Chrm.* 165 e f.; *Grg.* 451 a–e, 453 e). In the *Theaetetus*, number is divided into the odd and the even (198 a) and we find there also (174 a f.) a classification of numbers in terms of whether they are square (where a number is multiplied by itself) or oblong (where one number is multiplied by another).[5] Generally speaking, what is involved in this Platonic procedure is the introduction of a second level of abstraction over and above that of noetic numbers, which can themselves be considered as occupying a first level of abstraction with respect to sensible numbers, i.e. those numbers which have 'visible and tangible body' (*Rep.* 525 d). At this second level of abstraction, the *structure* of noetic numbers can be exhibited in terms of the possible arrangements of units and, on the Platonist account, it is only at this level that the generic classification and individuation can be made. The existence of this second level of abstraction is indicated by the fact that arithmetic and logistic are defined in the *Gorgias* and *Charmides* passages as being concerned with the odd and the even, and number itself is not mentioned.

Aristotle makes it clear that, unlike Plato, he does not accept that this second level of abstraction has any ontological implications: mathematical statements, he tells us, are 'not about anything over and above [spatial] magnitudes and numbers' (*Metaph. M*, 1077 b 18–19). Moreover, the attempt to provide a generic identity, and hence unity, for numbers is something that Aristotle clearly rejects on a number of occasions, and he explicitly denies that the Platonist view can tell us what makes a number *one* number (*Metaph. H*, 1044 a 2–6; *Λ*, 1075 b 34 f.; *M*, 1082 a 20 f.). In fact, he comes to reject any attempt to construct species out of a genus and its *differentiae* (*Metaph. Z*, 1038 b 24 f.; cf. also *de An. A*, 402 b 7; *P.A. A*, 643 b 9). This rejection of generic unity is completely general, extending even to political questions on the nature of the state, so that, for example, he rejects the Platonic idea that the community (κοινωνία) of the πόλις is 'one' (*Pol. B*, 1261 a 6 f.). As regards number, while he does say that odd/even, prime/composite and equilateral/oblong all belong to number (*APo. A*, 73 b 39–74 a 1), this must not be taken to indicate that these provide us with a classification which allows us to say of any number that it is 'one', for such an idea is clearly ruled out in *Metaphysics Δ*, 1021 b 1–8:

There is another way of speaking of quality and this applies to the objects of mathematics, as in the case of numbers which are said to have a certain quality, e.g. the composite numbers which are not in one dimension only but of which the plane and solid [numbers] are copies (these being those with two or three factors). And, in general, it means that which is in the essence besides

[5] Of some importance here is the use of the γνώμων, a geometrical configuration which is 'fitted' to formations of monads showing similarity of kind between these formations. For a discussion of figurate numbers and the γνώμων cf. T. L. Heath, *A History of Greek Mathematics* (2 vols. Oxford, 1921), i. 76–84, and his notes on Definition 16 of Book 7 of Euclid's *Elements* in his edition of *The Thirteen Books of Euclid's Elements* (3 vols. New York, 1956), ii. 287–90.

quantity. For the essence of each is what it is once, e.g. that of six is not what it is twice or thrice but what it is once, for six is once six.

Two questions must be distinguished as far as the generic construal of numbers is concerned. What Aristotle disputes is that a generic classification of number can provide criteria for the individuation of numbers, that it can provide criteria by which we can say that numbers are 'ones'. He is not disputing that numbers are subject to generic classification. His general conception of mathematics, as the first chapter of *Metaphysics E* makes clear, is of a unified theory dealing with whatever has no independent being and does not change. It is true that in the *Posterior Analytics* (75a39 f.) he speaks of geometry and arithmetic being generically distinct, but it is clear from other contexts that they are more properly considered to be subgenera of mathematics *per se*. In *Metaphysics E*, 1026a25–8, he speaks of geometry and astronomy being mathematical sciences which deal with different areas, while 'universal mathematics' deals with all mathematical areas. *Metaphysics K*, 1061 b 19 f. suggests that this may be a general science of quantity, which is exactly what we would expect, since quantity is one of the categories and hence a prime candidate for a *summum genus*. There can be little doubt that Aristotle sees numbers as generic concepts and indeed as predicates of quantity (cf. *Metaph.* 1020 a 7 f.), something which would have been in conformity with everyday usage and which Aristotle is, perhaps, intending to provide with a philological backing when he claims in *Physics Γ*, 207b8–10, that numbers (or, as we should say, number words) are paronymous: i.e. their use as nouns is derived from their use as adjectives. Because Aristotle always treats numbers, whether noetic or sensible, as being numbers *of* something (cf. *Ph. Δ*, 221 b 14–15), it is clearly more attractive to consider them as being *primarily* something which we predicate of something else (as adjectives) rather than vice versa (as nouns). This close tie between grammar and ontology is not, of course, peculiar to Aristotle. Frege, for example, who wishes to construe numbers as objects, goes to some lengths to give the logical form of number statements, even those in which the number words appear to be used attributively, as being one in which numbers figure only as 'terms', i.e. as proper names.

If Aristotle can allow that numbers are subject to generic classification, why cannot he allow that they are subject to generic individuation? There is certainly no obvious reason why their being properties should preclude their generic individuation. Consider the apparently analogous case of colours. Colours are also properties subject to generic classification, and just as white cannot be said to exist independently of something which is white (*Metaph. Z*, 1029 b 13 f.), so too ten cannot be said to exist independently of some ten things (*Metaph. A*, 990 b 17–22). Nevertheless, to distinguish colours and to distinguish things having those colours are quite different, and we are surely able to distinguish colours without distinguishing what has those colours; and if we do this we are distinguishing colours as such, i.e. generically. If this account is correct, and if numbers parallel colours in this respect, then there is something seriously wrong with Aristotle's account of number. Number words, it is true, are always collective predicates whereas colour words are normally distributive predicates, but this has no bearing on the status of numbers and colours as generically classifiable properties. If we do draw parallels in the case of numbers, then we should be able to say that we can distinguish between numbers without invoking what they are the numbers of, and to say this is tantamount to saying that we can individuate numbers generically. The reasons why Aristotle denies that we can so individuate numbers are, I suggest, highly cogent, but the question can only be clarified by reference to his general doctrine of what is required in order that something be 'one', so it is to this doctrine that we shall now turn.

II. INDIVIDUATION IN GENERAL

Aristotle's account of number hinges on the fact that in counting we can only count things of the same kind, and on the fact that this requires prior differentiation and grouping together of the things to be counted. This is a reasonably straightforward principle, which is surely fundamental to any theory of counting, and which we may call the homogeneity requirement for counting. This requirement clearly rules out the idea that unity could be conferred on a collection simply in virtue of our ascribing a number to it, since homogeneity is a precondition of counting, not its consequence. But this still leaves open the question of how Aristotle conceives of homogeneity. His interpretation of the homogeneity requirement is, in fact, rather complicated. He considers that what supplies the homogeneity required for counting is the 'one' which is the 'measure' (*μέτρον*) by which we count, for number is a plurality 'measured' by a one (*Metaph. I*, 1057 a 3–4): number and the one are as measured and measure respectively (*Metaph. I*, 1057 a 5–6; 1052 b 22 f.). Moreover, because 'one' is the principle (*ἀρχή*) of number or, as we might now say (glossing slightly), a condition of possibility of ascribing number, it cannot itself be a number (*Metaph. N*, 1088 a 6–8).

This argument is peculiar, but the peculiarity cannot be put down to a linguistic confusion brought about by ambiguities in the expression *τὸ ἕν* in Greek. It is true that *τὸ ἕν* covers 'one', 'unit', 'unitary', 'unity' and so on, and this is not merely a lexical nuance that can be overlooked, particularly since Aristotle is often concerned to elucidate philosophical problems by analysing common forms of speech (cf. *Top. Z*, 147 b 20–2 and, on *τὸ ἕν*, *Metaph. Δ*, ch. 6). But this procedure does not render him a victim of his language, something about which he is very careful in mathematical contexts (e.g. *Metaph. I*, 1053 a 25–30). Moreover, the problem is not so much linguistic as mathematical, and it is a problem shared by all Greek writers on mathematics for, although we do occasionally find 'one' being used as if it were a number (e.g. Plato, *Soph.* 238 b, and in Aristotle, *Metaph. M*, 1080 a 30–5), it is never theorized as such. The Eleatics and Pythagoreans treat one as falling outside the realm of number, Plato does so (*Phd.* 103–5) and, after Aristotle's time, we find Euclid, in the *Elements*, proving an arithmetical theorem separately for one (Book VII, prop. 15) and for number (Book VII, prop. 9). The idea that 'one' is a number was rejected in the main on the basis of the highly intuitive conception of number as a plurality, combined with a typically Eleatic form of *reductio ad absurdum*. The fact that number is defined as a plurality of units entails that if one were a number it would be just such a plurality; but it cannot be both one and a plurity, therefore it cannot be a number.[6]

Aristotle's doctrine of *τὸ ἕν* raises a serious problem concerning the 'one' by which we count. It is tempting to see the one by which we count as being the kind whose members are counted: to consider that it is the sheep's being one in kind, for example, that enables us to say that they are ten. In so far as this view suggests that, in counting, we start with a kind and then individuate and count the members of that kind, it is highly misleading, since this is exactly contrary to Aristotle's intentions. Of some importance here is the fact that when giving examples of counting Aristotle invariably chooses animals. Animals are each one in virtue of their invidiual *ψυχή*, and this constitutes their distinct form (cf. *Metaph. Z*, 1035 b 14–31, 1037 a 5–33; *H*, 1043 a 35–6; *M*, 1077 a 21–34). Although the doctrine that every substance has a particular form of its own is developed into a general theory covering inanimate substances as well (e.g. *Metaph. Λ*, 1070 a 22 f. and *Z*, ch. 13), animate things are paradigmatically individuated in virtue of their forms. They are less problematically so individuated

[6] cf. A. Szabó, *The Beginnings of Greek Mathematics* (Dordrecht, 1978), pp. 257 f.

than inanimate things, and Aristotle commits himself to animate things having particular forms much more often than he does to inanimate things having such forms.[7] In the light of this, there are two reasons why Aristotle's choice of example is significant.

First, the fact that animals are paradigm substances means that they are paradigm 'ones'. Unlike heaps of matter, or qualities such as colour, or periods of time, they can be indicated demonstratively, since they form wholes distinct from their parts. It is such a consideration that Aristotle appears to have in mind when he speaks of individuals being 'one in number' at *Metaph. I*, 1052a31–2.[8] To say that substances are paradigmatically 'ones' is, however, not to say that they are the only things which are 'ones'. It is true that Aristotle denies that the parts of living things are distributively 'ones', for example at *Metaph. Z*, 1040b5–15, but we should not take him to mean that I can neither identify nor count arms or legs, since he makes it clear in the subsequent sentence that 'one' is used like 'being' (1040b16), an idea that he restates later in the *Metaphysics* when he says that τὸ ἕν and τὸ ὄν are convertible (*K*, 1061b16 f.). This does not mean that the focal meanings of τὸ ἕν and τὸ ὄν are the same, for they are not: the focal meaning of τὸ ὄν lies in the category of substance whereas that of τὸ ἕν lies in the category of quantity (cf. *Metaph. I*, 1052b20–5). But this difference in focal meaning does not preclude the paradigm instances of 'ones' being individual substances. To say that individual substances are paradigm 'ones' is not to say that other 'ones' are logically dependent upon these or that 'one' can only be defined by reference to substance, only that whatever else we count as 'ones' we must always include individual substances. Aristotle does occasionally allow that things which are merely 'one' by continuity, like the *Iliad*, or merely 'one' by being tied together, are each still 'one', but they cannot be 'one' in either the primary focal sense or in the paradigmatic sense. Similarly, the parts of living organisms are not 'ones' in either of these senses, but they can nevertheless be considered to be 'ones' in a derivative way, which is why they can be counted.

Secondly, as ch. 13 of *Metaphysics Z* makes clear, Aristotle considers that substances are essentially, and not merely conventionally, individual. We generally tend to think of the identification of particulars in terms of a logically prior application of a predicate which allows us to individuate such particulars and group them together in terms of an attribute that they have in common, where this attribute is designated by the predicate that we have chosen. On this conception, we are relatively free to choose how we 'carve up the world', since the pertinent distinguishing features of particulars will depend upon how we choose to characterize them, and there may be a large range of such characterizations adequate for our purposes. Aristotle would, I think, allow this to a limited extent in the case of identifying things which involve a mixture of categories, such as 'musical men' or 'white cloaks' (cf. *Metaph. Δ*, 1015b16 f. and *Z*, 1029b13 f.), but such identification is dependent upon our first being able to identify the substances involved, and it is here that the question of substances being essentially individual arises. As far as substances are concerned, the world is already 'carved up', so to speak, and what we must do is to recognize and articulate the way in which it is already carved up. An individual substance is not individuated

[7] Considerable problems arise when the doctrine of particular substances is extended to cover inanimate substances, and the topic has been the subject of considerable debate. Much of the literature on the question is cited and discussed in E. D. Hartner, 'Aristotle on Primary *ΟΥΣΙΑ*', *Archiv für Geschichte der Philosophie*, 57 (1975), 1–20.

[8] cf. L. Elders, *Aristotle's theory of the One* (Assen, 1961), pp. 65–7, for a detailed discussion of Aristotle's use of this expression.

on the basis of a predicate that applies to it; rather, this predicate can be said to apply to it only because it already possesses a certain form. It is because individual substances already each have a particular form which naturally differentiates them from other things that they are individual in the first place: the possession of a form by each individual substance is logically prior to the differentiation of such substances into a plurality.

These two features of individual substances – the fact that they are paradigmatically ones and the fact that they are essentially individual – have an important bearing on the status of the 'one' or 'measure' by which each class or kind of things is known (*Metaph. I*, 1052b20–5) and by which each plurality is measured (*Metaph. I*, 1057a3–4). Because a kind is logically posterior to the things making up that kind, it is the individual that is the measure of the kind, and not the kind that is the measure of the individual. The species 'dog', for example, is not something over and above individual dogs, and hence the dogs are not one in virtue of being of the same species. Similarly, a number is not something over and above its units (cf. *Metaph. M*, ch. 3).[9] This is not to deny that there are collections of things which have something in common, but rather to stress the fact that what they have in common is not something over and above the things themselves, like a Platonic Form. We can say of ten dogs that they are a determinate plurality and not an amorphous mass, not because we have chosen to characterize and individuate them in some purely conventional way, but because they are essentially individual substances which are similar to one another in virtue of something which is constitutive of the essence, namely their forms (in this case their individual ψυχαί). In short, it is because the dogs are already individuated *in nature* that we can say that they have something in common, and it is in virtue of this that we can treat them as being of the same kind, and as being a *determinate* plurality.

III. THE INDIVIDUATION OF NUMBERS

The measure of a sensible number is a sensible object. In the paradigm case this sensible object is a substance: the 'one' by which we measure a decad of dogs, for example, is the individual dog. Noetic numbers, on the contrary, are by definition not numbers of sensible objects so their 'measure' cannot be a sensible object. Now kinds, sensible numbers and noetic numbers have one thing in common: they are all properties, and hence none of them can be said to have an independent existence or 'being'. It is true that we can define mathematical attributes independently of the particular sensible things possessing those attributes, whereas kinds cannot be so defined (*Ph. B*, 193b31–194a7). But matter must be part of any definition (*Metaph. H*, 1045a34–5), and this applies not only to kinds and sensible numbers but also to noetic numbers. Since it is sensible objects that provide the measure of sensible numbers, we may ask what it is, if anything, that provides the measure of noetic numbers: what, if anything, is the noetic correlate of a sensible object?

The answer to this question lies in Aristotle's doctrine of abstraction (ἀφαίρεσις). When mathematical attributes such as numbers are defined in terms of the physical objects possessing those attributes they clearly have sensible matter as their matter. When they are defined independently of such objects their matter is what Aristotle terms ὕλη νοητή, noetic or 'intelligible' matter (cf. *Metaph. Z*, 1036a9–12). Mathematical abstraction is distinctive in that it is a twofold process: we must abstract the

[9] Aristotle provides the same kind of account for infinity (*Ph. Γ*, 204a8–206a8), place (*Ph. Δ*, 209b1–210a13) and time (*Ph. Δ*, 218b21–220a26).

mathematical properties of the object or collection by disregarding what it is that has those properties (i.e. the matter), but there is also a second part to the abstraction in which we disregard the *properties* of sensible objects so that what has these properties becomes the object of investigation. These two parts of the abstraction taken together yield mathematical properties and a noetic matter of which these are the properties. In abstracting numbers we 'detach' them from sensible things, but it is an essential characteristic of numbers (and geometrical figures) that they be properties, so we must therefore 'attach' them to something else; otherwise they would be 'free floating' properties, so to speak, and this is as impossible in the case of numbers as it is in the case of kinds.

The idea of noetic matter is introduced by Aristotle in the context of geometry, and his argument is intended to secure that the geometrical properties that are abstracted from sensible bodies do not thereby become independent.[10] It secures this by introducing the second kind of abstraction mentioned above, whereby we disregard the properties of sensible objects and are left with a propertyless but extended substratum upon which we then impose our abstracted geometrical properties. In this way, the essentially attributive nature of figures is maintained. As regards the extended substratum that we abstract, this can be of one, two or three dimensions, depending upon whether we are dealing with linear, plane or solid figures. Parallel considerations apply in the case of arithmetic, since Greek arithmetic is conceived on the basis of a metrical geometry, but there is a crucial difference: in arithmetic we are no longer dealing with (continuous) lines, planes and solids but with (discontinuous) determinate sums of unit lengths, unit areas and unit volumes.[11]

The idea of noetic or intelligible matter enables us to solve our two main problems: the problem of the parallels between colour and number, and the problem of individuation. We noted above that there is an apparent anomaly in the treatment of colours and numbers in that, while both are subject to generic classification, the former appear to be subject to generic individuation whereas Aristotle claims that the latter are not; we also failed to find a relevant difference between the two to explain this. In fact, the anomaly is more apparent than real, since colour is subject to the same problems as regards generic individuation as number is.

We can distinguish between three different uses of number words, figure words and colour words. The first is the straightforward adjectival use where the words function predicatively, both grammatically and (for Aristotle) logically. The second is the use of an abstract noun formed from the corresponding adjective, and the third is the genuinely substantival use. In the case of the colour red, for example, the abstract noun would be 'redness' and the substantive would be 'red'. It is 'red' that we consider to be the primary colour complementary to 'green', and not 'redness'; conversely, it is the peculiar quality of 'redness' that we might admire in a painting of a sunset by Turner, not the peculiar quality of 'red'. The distinction between abstract and substantive usages that I want to emphasize here is a conceptual one: the lexical and etymological differences are helpful but not necessary to the distinction. The abstract usage is dependent upon the predicative usage, whereas the substantive usage is relatively independent and indicates that colours are being taken seriously as at least second-order, and perhaps first-order, objects in their own right.

In the case of number, it would clearly not be conducive to Aristotle's treatment

[10] A good account of Aristotle's view of the role of noetic matter in geometry is given in I. Mueller, 'Aristotle on Geometrical Objects', *Archiv für Geschichte der Philosophie*, 52 (1970), 156–71.

[11] I have defended this interpretation of the noetic matter of numbers in my 'Aristotle on Intelligible Matter', *Phronesis*, 25 (1980), 187–97; it would take us too far from our main topic to enter into the details of the justification here.

of noetic numbers to construe them as objects designated by substantives. He makes the point himself when he maintains that number words are paronymous: this would be a peculiar remark to make unless he had in mind this kind of issue. To allow otherwise – to allow that number words are substantives – would be tantamount to the reification of properties of which he so often accuses Plato. It is in order to avoid such reification in mathematics that the doctrine of noetic matter is introduced, but it is clear that similar considerations should apply equally in the case of colours. When we classify numbers generically (as at *APo.* 73b39–74a1) and colours generically (as in *Sens.* ch. 4), we are classifying properties which must be attached to something. Since these properties are not attached to sensible matter we can, I suggest, speak of noetic colours just as we can speak of noetic numbers or noetic figures. To think of (in our sense of 'imagine') a colour is to think of a coloured expanse, just as to think of a triangle is to think of a triangular expanse. This 'expanse' which has the property of being a particular colour must, moreover, be material since, as Aristotle's arguments against the void bear out, properties must always be attached to matter: there can be no properties where there is no matter.

As far as the question of individuation is concerned, noetic numbers have two features which serve to distinguish them from sensible numbers: they are potential and they are equivocal, whereas sensible numbers are actual and univocal. They are potential (cf. *Metaph. M*, 1084b20–3) because though they are objects of knowledge actual knowledge is always of the individual and never of the general: sight, for example, is directly of a particular colour and only 'incidentally' of colour in general, just as grammatical knowledge is directly of a particular alpha and only 'incidentally' of alpha in general (*Metaph. M*, 1087a10–25). We can picture noetic numbers as line lengths,[12] where the line is a one-dimensional noetic substratum. On this conception, the noetic 'ten' would be this line length divided into ten parts. If the line length were actual, if it were a sensible line length, we would actually have ten parts, but since we are dealing only with an abstraction, with the line length in general, the ten parts are merely potential and to actualize the ten parts we would have to divide some given sensible line into the corresponding parts. In this sense, the unit by which we imagine the noetic line length to be divided stands for that by which we divide the sensible line length and, more generally, for the individual sensible object which is the 'measure' of a decad, such as the individual dog in a decad of dogs.

This brings us to the question of the equivocality of noetic numbers, which is best introduced by way of an example. In the *Physics* (*Δ*, 224a2–16), Aristotle points out that a decad of dogs and a decad of sheep are different decads, but that 'they have the same number'. That they are different decads is clear from the fact that the 'one' or 'measure' of the first is a dog whereas that of the second is a sheep, and because the 'ones' or 'measures' are different the decads are different. Now 'one' is used here equivocally, and at *Physics H*, 284b19–21, the fact that 'one' is equivocal is taken to entail that 'two' (and, by extension, presumably any number) is equivocal.[13] Note, however, that what is equivocal here is the *noetic* number. The sensible numbers, the

[12] This is the way in which numbers are usually notated in Greek arithmetic (cf. Books 7 to 9 of Euclid's *Elements*) and it is upon these line lengths that arithmetical operations are performed. In multiplication, for example, the product of two lines is a rectangle having those lines at its sides. This procedure places severe conceptual and computational constraints on Greek arithmetic as compared to the arithmetic developed from the seventeenth century onwards: cf. M. S. Mahoney, 'The Beginnings of Algebraic Thought in the Seventeenth Century', in S. Gaukroger (ed.), *Descartes: Philosophy, Mathematics and Physics* (New Jersey, 1980).

[13] This point is different from Aristotle's more specific remark at *Metaph. I*, 1053b25–1054a10, where he argues that 'one' has different uses in different categories. What we are concerned with here is the equivocality of 'one' as applied to two things of the same category (substance).

two decads, are not equivocal because their 'ones' are not equivocal. Our knowledge of numbers is *directly* of these decads, each having its own 'one' or 'measure' and each, in virtue of this, having its own 'ten'. In knowing such decads we also have a derivative or 'incidental' knowledge of the 'ten' which the decads have in common, but what makes each decad a decad is a 'one' peculiar to it, and it is in virtue of these 'ones' that the decads have the same number, *not* in virtue of some 'one' peculiar to the noetic 'ten' and transcending sensible 'ones'. The noetic number has no 'one' of its own, and it is precisely because it can take *any* 'one' that it is equivocal. Indeed, this is part of the reason why it is only potential, because it must take a 'one' to be actual.

In this sense, number words are like other words designating magnitudes. Two things can be large but by different measures, in which case 'large' would be used equivocally, but the equivocation does not prevent them both being called large. Similarly, the sheep and dogs are ten by different measures, but they are both still *ten*. (If we were counting animals, or substances in general, then they would be ten by the same measure of course, but in this case we would be able to include our sums in the same total, something which we cannot do when we use different measures.) We do not have a general absolute criterion for something's being large because whether something is large or not can only be decided with respect to some unit. Once we have chosen the unit, however, the problem disappears. Similarly with number: we need a 'one' to render any plurality determinate and it is therefore only when we have a 'one' that counting can begin. There is no general absolute criterion, independent of our providing a specific 'one', by which to distinguish different numbers.

IV. A RESIDUAL PROBLEM: NUMBERS AS PROPERTIES

It is on the fundamental question of whether numbers are properties that Aristotle parted company with Plato, but it is also on this very issue that Frege was to part company with Aristotle (and those who held more modern versions of the thesis such as Mill). Frege's very cogent arguments, in his *Foundations of Arithmetic*, against the thesis that numbers are properties are well known. I shall not consider here the arguments designed to show that the thesis is subject to special and irresoluble difficulties in the case of nought and one, since Aristotle, in accord with the common practice of his time, does not consider nought and one to be numbers: indeed, Frege's arguments could be taken to support Aristotle's position in that they could be taken to show that a predicative account of number precludes one and nought from being numbers. Nor shall I consider the argument that the logical form of number statements shows that numbers are not predicative, partly because Aristotle's logical apparatus could not register such a difference, and partly because there are in fact ways of giving the logical form of number statements such that they do turn out to be predicative.[14] The argument which is of most interest to us is that concerning the differences in conditions for ascription of numbers and properties. This argument is, to my mind, the most compelling of Frege's objections, and I think it shows the strength of Aristotle's position that it provides us with a way round the objection.

Frege considers genuine properties, such as colours, to be independent of how we choose to regard the object to which we ascribe the property, whereas in the case of

[14] cf. C. H. Lambros, 'Are Numbers Properties of Objects?', *Philosophical Studies*, 29 (1976), 381–9. Lambros presents what is, I think, a good *prima facie* case against Frege; whether it is conclusive is a different matter altogether, and one that I shall not take up here.

number it is clear that what number we ascribe to something depends upon how we choose to regard it; indeed, we cannot ascribe number to something unless we choose to regard what we are counting in some way. For Frege, this means that numbers are not properties which objects actually have, and this is part of the reason why we can never identify things on a numerical basis. We can pick things out in virtue of their having a particular shape, or a particular colour, etc., but we can never pick them out in virtue of having a particular number. Shape and colour enable us to pick things out because things actually have particular shapes and particular colours, but they do not have particular numbers. The ascription of number requires a choice of characterization on our part, and the choice may be quite arbitrary.

It is an open question whether Aristotle was fully aware of the problems here, but it is of interest that his examples, which are always of paradigmatically and essentially individual substances, in fact circumvent the problems by introducing cases where, he thinks, *the choice is effectively made for us by nature*. This is a consequence of his theory of particular substances, and how we assess his conception of number will therefore depend, in the final analysis, upon our assessment of his view of individual substances. Given this view, however, there can be little doubt that his account of number is a coherent one.

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